

Week 2 Worksheet

Identical Particles

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Exercise Warm up. Suppose you had three particles, and three distinct one-particle states are available, ψ_a , ψ_b , and ψ_c . How many different three-particle states can be constructed if they are (a) distinguishable particles, (b) identical bosons, or (c) identical fermions? (The particles don't have to be in *different* states—if they are distinguishable, we could have $\psi_a(x_1)\psi_a(x_2)\psi_a(x_3)$ as an allowed state, for example).

Exercise 1. a) Write down the hamiltonian for two noninteracting identical particles in the infinite square well. Write down the ground states for the three cases: distinguishable, fermions, bosons. Recall that the one-particle wavefunctions are

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right),$$

with energies $E_n = n^2\pi^2\hbar^2/2ma^2$.

b) Find the first three excited states and their energies for each of the three cases (distinguishable, fermions, bosons).

Exercise 2. In Exercise 1, we ignored spin (or at least supposed that the particles are in the same spin state).

a) Do it now for particles of spin 1/2. Construct the four lowest-energy configurations, and specify their energies and degeneracies.

b) Do the same for spin 1 (you will need the Clebsch-Gordan table from bCourses).